

**East Africa Collaborative Ph.D. Program  
in Economics and Management**

**Income Distribution and Economic Growth:  
Empirical Evidence from an Evolutionary  
Growth Perspective**

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## **Preface**

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# **Income Distribution and Economic Growth: Empirical Evidence from an Evolutionary Growth Perspective\***

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## **Abstract**

This paper links access to bank loans and income distribution to productivity growth. The main focus of the paper is examining how functional income distribution can influence the evolution of productivity and thereby promote economic growth. We obtained key variables and their evolution from the Central Statistical Agency (CSA) dataset. We employed Nelson and Winter's (1982) evolutionary economic framework; the evolutionary theory of economic change and subsequent developments are used jointly with an evolutionary econometric approach which sees economic growth as an open ended process. The major conclusion of this paper is lack of strong evidence of evolution (intra-industry selection) to foster productivity growth and re-allocation (structural change).

Keywords: Functional income distribution, economic growth, evolutionary economics, complexity, evolutionary econometrics, logistic diffusion.

JEL Classification Codes: C60; C600; C63.

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## 1. Introduction

The question of how inequality is generated and how it evolves over time has been a major concern of economics for more than a century. Yet the relationship between inequality and the process of economic development is far from being an agreed area of research. In developing economies, it is a challenge for both academic and policy circles. There is a demand for academicians to investigate this and it is an issue that needs to be dealt with by policymakers.

Thus, the study of income distribution should not be undertaken for the sake of study but for its wider implications on economic performance. One aspect of economic performance that is affected by it is economic growth because its growth inequality linkage is both important and controversial.

It is important because policymakers need to understand the way in which an increase in output will be shared among different groups within an economy and the constraints that this sharing may put on future growth. Its controversial aspects arise from the fact that it has been difficult to reconcile the different theories, especially since empirical evidence has been largely inconclusive (Cecilia, 2010). For example, Barro (1990) and Persson and Tabellini (1994) argue that moderate redistribution promotes growth whereas a high degree of redistribution will have a negative impact on growth.

On the effect of inequality on growth, the conventional textbook approach is that inequality is good for incentives and therefore good for growth, even though incentive and growth considerations might be traded off against equity goals. On the other hand development economists have long expressed counter-arguments.

For example, Todaro (1997) provides four general arguments why greater equality in developing countries may in fact be a condition for self-sustaining economic growth: (a) dissaving and/or unproductive investments by the rich; (b) lower levels of human capital held by the poor; (c) demand pattern of the poor being more biased towards local goods; and (d) political rejection by the masses.

Overall, the view that inequality is necessary for accumulation and that redistribution harms growth has faced challenges from many fronts. For example, Alesina and Rodrik (1994) and Persson and Tabellini (1994), combine political economy arguments with the traditional negative incentive effect of redistribution. These authors maintain that inequality affects taxation through the political process when individuals are allowed to vote in order to choose the tax rate (or, equivalently, vote to elect a government whose programs include a certain redistributive policy). If inequality determines the extent of redistribution, it will then have an indirect effect on the rate of growth of the economy.

In their paper 'Social Conflict, Growth and Income Distribution', Benhabib and Rustichini (1996) explore the effect of social conflict arising due to income distribution on both short-run and long-run economic growth rates. According to them, despite the predictions of the neo-classical theory of economic growth, poor countries were observed to invest at lower rates and have not grown faster than rich countries. They studied how the level of wealth and the degree of inequality affects growth and showed how lower

wealth can lead to lower growth and even to stagnation when the incentives to domestic accumulation are weakened by redistributive considerations.

Perotti (1996) contends that equality has a positive impact on growth while Rehme (2006) argues that redistributing governments may have a relatively stronger interest in technological advances or high economic integration. He observes a positive association between redistribution and growth across countries.

While we can find vast literature on income inequalities and economic growth similar to the ones motioned earlier, they exclude the role of firms and the mechanisms behind them for the creation and evolution of the links between income distribution and economic growth. However, the existence of firms and their actions are recognized in economic theory.

Thus, our introduction of firms into such an analysis is not arbitrary. Firms play a central role as sources of growth and in the economic evolution process. This argument is theatrically consistent with one of the questions in economics (Coase, 1937). Thus, any analysis which omits the role of firms in the creation and evolution of income distribution in the growth process cannot make a complete description. More specifically, empirical evidence on how firms' financial structures can influence their productivity and thereby drive economic growth is scarce. This study tries to bridge this gap.

Two crucial questions arise for policymakers which have policy relevance. The first is whether inequality is a pre-requisite for growth. And the second concerns the effects of growth promoting policies on inequality, and in particular under which circumstances a conflict between the two objectives may emerge.

Thus, this paper takes firms as a hub for generating macroeconomic regularities. Firms generate link between sources and uses of funds, productivity, income distribution and structural transformation in the market process. We explore the dependence of macroeconomic productivity growth on firm-level productivities. We examine how firms' access to bank loans can influence an aggregate rate of growth. The growth of productivity, output and employment are determined mutually and endogenously. More specifically, this paper answers the following questions:

- How do firm level sources and uses of funds (investments from bank loans) influence economic growth?
- Does access to bank loans affect intra and inter-firm reallocation of labor?
- Can we find evidence of structural change, that is, reallocation of labor from less productive to more productive industries?
- Can we draw some theoretical results and what policy lessons can we draw from this?

The rest of the paper is organized as follows. Section 2 is an excursion into economic growth theories. Section 3 deals with evolutionary economics and economic growth from an evolutionary perspective. Section 4 deals with econometric modeling in the presence of evolutionary change; it also presents empirical evidence and is followed by Section 5 which presents empirical results from Ethiopia. Section 6 gives a conclusion.

## **2. Theory of economic growth**

Economic growth is a dominant area of theoretical and empirical research in economics in general and in macroeconomics in particular. For example, Nelson (1996: 7) points out that from the beginning of modern economics as a field of study, economic growth has often been the central area of inquiry, but on and off. During the early decades, Hahn and Matthews (1964) presented the most comprehensive survey on the contributions that had been made to the theory of economic growth beginning with Harrods's article in 1939. Salavadori (2003) emphasizes that an interest in the study of economic growth has experienced remarkable ups and downs in the history of economics. It was the central issue in classical political economy from Adam Smith to David Ricardo, and then in the critique by Karl Marx (Nelson, 1996; Salavadori, 2003).

The growth theory waned (Nelson, 1996), moved to the periphery during the so-called marginal revolution (Salavadori, 2003). Undoubtedly one of the reasons for this was that formal theory had developed which focused on market equilibria. The concern was with what lay behind demand and supply curves and how these jointly determined the observed configuration of outputs, inputs and prices. The troubled economic times after World War I, in particular the great depression, also tended to pull the attention of economists towards analyzing shorter-run phenomenon such as balance of payments disequilibria, inflation and unemployment.

There was a renaissance of interest in long-run economic growth after World War II. One reason for this was that the new national product data was first available for United States, and later for other advanced industrial nations. This for the first time allowed economists to measure economic growth at the national level (Nelson, 1996).

In modern times, the starting point for any study of economic growth is the neo-classical growth model which emphasizes the role of capital accumulation. This model, first constructed by Solow (1956) and Swan (1956), shows how economic policy can raise an economy's growth rate by inducing people to save more. But the model also predicts that such an increase in growth cannot last indefinitely. In the long run, a country's growth rate will revert to the rate of technological progress, which neo-classical theory takes as being exogenous. Underlying this long-run result is the principle of diminishing marginal productivity, which puts an upper limit on how much output a person can produce simply by working with more and more capital given the state of technology. Aghion and Howitt (1992, 1998) provide a splendid presentation on this.

### **2.1 The neo-classical growth theory**

In the neo-classical framework, the notion of growth as increased stocks of capital goods was codified as the Solow-Swan growth model, which involves a series of equations that show the relationship between output, labor-time, capital and investment. This was the first attempt to model long-run growth analytically. According to this theory, the role of

technological changes became crucial and even more important than the accumulation of capital.

It assumes that countries use their resources efficiently and that there are diminishing returns to capital and labor. From these two premises, the neo-classical model makes three important predictions: first, increasing capital relative to labor creates economic growth, since people can be more productive given more capital. Second, poor countries with less capital per person grow faster because each investment in capital produces a higher return than in rich countries with ample capital. Third, because of diminishing returns to capital, economies eventually reach a point where any increase in capital no longer creates economic growth.

The model also notes that countries can overcome this steady state and continue growing by inventing new technology. In the long run, output per capita depends on the rate of saving, but the rate of output growth should be equal to any saving rate. In this model, the process by which countries continue growing despite diminishing returns is 'exogenous' and represents the creation of new technology that allows production with fewer resources. As technology improves, the steady state level of capital increases and the country invests and grows.

The strengths of the neo-classical approach for economic growth are considerable. Neo-classical theory has provided a way of thinking about the factors behind long-run economic growth in individual sectors and in the economy as a whole. The theoretical structure has called attention to historical changes in factor proportions and has focused an analysis of the relationship between those changes and factor prices. These key insights and the language and formalism associated with them have served effectively to guide and to give coherence to research that has been done by many different economists around the globe. The weakness of the theoretical structure is that it provides a grossly inadequate vehicle for analyzing technical change.

The fundamental problems with neo-classical explanations of economic growth are that: (1) despite much empirical efforts at the neo-classical production function, the model still faces problems in explaining considerable inter-plant and international differences in productivity as well as differences between developed economies. Even more striking is evidence for single industries, showing big sectorial productivity gaps between different countries (Hodgson, 1996) and (2) increasing capital creates a growing burden of depreciation. It is also noted that the economic life of capital assets has been declining. In particular, the orthodox formulation offers no possibility of reconciling analyses of growth undertaken at the level of the economy or the sector with what is known about the processes of technical changes at the microeconomic level. Hodgson (1996) has a detailed account of this and similar arguments.

## **2.2. Endogenous growth theory**

In response to some of the problems in the standard neo-classical growth theory, the idea of an endogenous growth theory emerged in the works of Romer (1986, 1987, 1990, 1994), Lucas (1988) and a second generation variant pioneered by Aghion and Howitt (1992,

1998). They developed the endogenous growth theory which includes a mathematical explanation of technological advancement.

This broke from the preceding neo-classical thinking by encompassing learning by doing and knowledge spill-over effects. In these models, cumulative divergence of national output and productivity becomes more likely than convergence and thus seems to correspond more adequately to available data.

However, the amended aggregate production function is still at the conceptual foundation of the endogenous growth models, typically embodying features such as increasing marginal productivity of knowledge but diminishing returns in the productivity of knowledge (Hodgson, 1996).

Therefore, overall there are constant returns to capital and economies never reach a steady state. Growth does not slow as capital accumulates, but the rate of growth depends on the type of capital that a country invests in. Research done in this area has focused on what increases human capital (for example, education) or technological change (for example, innovation).

### **3. Economics as an evolutionary science and economic growth from an evolutionary perspective**

#### **3.1 Why an evolutionary approach in economics?**

Evolutionary theory in economics is as old as economics itself. It was pioneered by Veblen (1898) when he asked, ‘Why is economics not an evolutionary science?’ and suggested that the only rational approach for economists was to assume economies to evolve. Otherwise, he argued, we can describe economy but have no effective theory of change and development. Veblen started his argument by asserting that all modern sciences are evolutionary sciences (p. 374) and Boulton (2010) reinforced Veblen’s suggestion by stating that ‘evolutionary economics is the only rational proposition’.

The renaissance in evolutionary economics in the past two decades has brought with it a great deal of theoretical developments and interdisciplinary import (Dopfer and Potts, 2004).

Inspired by the Veblen’s theory, evolutionary economics has become one alternative approach to economic analysis involving complex economic interactions. Recent contributors include Nelson (1974), *Neoclassical vs Evolutionary Theories of Economic Growth: Critique and Prospectus*. More importantly, Richard Nelson and Sidney Winter’s seminal work *An Evolutionary Theory of Economic Change* (1982), Dopfer’s *The Evolutionary Foundations of Economics* (2005) and Beinhocker’s *The Origin of Wealth, Evolution, Complexity, and the Radical Remarking of Economics* (2006) are recent advancements in the theory of evolutionary economics.

The questions to be answered before using an evolutionary theoretical framework to understand how economies grow are: What is evolutionary economics? Why evolutionary economics? What are the theoretical foundations of evolutionary economics? Where do

economies come from? (Beinhocker, 2006). How do the behaviors, relationships, institutions and ideas that underpin an economy form, and how do they evolve over time?

Beinhocker has argued that questions about origins play a prominent role in most sciences because as it would be difficult to imagine modern cosmology without the Big Bang, or biology without evolution, it would be hard to believe that economics could ever truly succeed as a science if it were not able to answer the question ‘Where do economies come from?’

Yet the question of the origin of economies has not played a central role in traditional economics which has tended to focus on how an economy’s output is allocated rather than how it got here in the first place. The process of economy formation presents us with a first-class scientific puzzle and one of the sharpest distinctions between traditional economics and what is described as Complexity Economics (Beinhocker, 2006).

But what is evolution in economic science? A relatively narrow definition of evolution is by the change in the mean characteristics of a population (Andersen, 2004). Economic growth, that is, the aggregate change in real output per person, is a consequence of increasing the productivity of the factors of production and of technological change in a very wide sense. For a constant participation rate, it can be modeled as a change in firm-level mean real output per employee weighted by the firm’s employment share in the population of firms in the economy. In Holm (2014) this is referred to as the evolution of labor productivity.

The key ideas of evolutionary theory are that firms at any time are viewed as possessing various capabilities, procedures and decision rules that determine what they do given external conditions. They also engage in various ‘search’ operations whereby they discover, consider and evaluate possible changes in their ways of doing things. Firms, whose decision rules are profitable, given the market environment, expand; those firms that are unprofitable contract. The market environment surrounding individual firms may be in part endogenous to the behavioral system taken as a whole; for example, product and factor prices may be influenced by the supply of output of the industry and the demand for inputs (Nelson and Winter, 1982).

According to Holm (2014), economic evolution is an open-ended process of novelty generation and the reallocation of resources. Selection is the sorting of a population of agents (firms) that is implicit to their differential growth rates. Firms perform innovations and develop knowledge in attempts to gain decisive competitive advantages over competitors, but firms are only intentionally rational agents with limited information and innovation, or more generally, learning may thus also lead to decreased productivity. Firms prosper or decline as a result of the interaction between their own learning activities, the learning activities of competitors and the external factors that set the premises for the interaction. We can find more on this in Dosi and Nelson (2010) and Metcalfe (1998). Safarzyńska and (2010) is also an excellent survey.

Holm (2014), explores how the evolution of productivity or any other characteristic in a population of firms can be described. According to him, evolution can be understood as the sum of two effects, which is referred to by different names in literature: inter-firm or

reallocation or selection effect and intra-firm or learning or innovation effect. To this, the effects of entry and exit are added but as far as entry is the introduction of new knowledge by entrepreneurs and exit is the disappearance of an inferior firm, these effects are also learning and selection. As a stylized depiction of economic evolution Holm (2014) expressed evolution as the total effect of selection, learning, entry and exit.

Whereas inter-firm selection is driven by the process of competition, inter-industry selection is driven by the process of structural change, which is somewhat different. Productivity understood as physical efficiency is important in competition among firms which produce homogenous products, for example, within industries. This is less the case with heterogeneous outputs because computing physical efficiency for heterogeneous products does not make sense because as the composition of demand changes over time, not least as a consequence of economic growth in itself, relative prices change as well, and this affects inter-industry selection (Holm, 2014; 1012).

Holm has emphasized the importance of indicating the basic differences between standard growth theories and growth theories in evolutionary economics. Evolutionary economists (for example, Richard Nelson, Eric Beinhocker, Geoffrey Hodgson and John Foster) strongly argue that an evolutionary framework is more encompassing than standard approaches. Carlsson and Eliasson (2003) note that economic growth can be described at the macro-level and never explained at that level. Economic growth is basically a result of experimental project creation and selection in a dynamic market and in hierarchies, of the capacity of the economic system to capture winners and losers. Castellacci (2007) gives an excellent review on the evolution of evolutionary theories in economics which is presented in Table 1.

Metcalf and Foster and Ramlogan (2006) explored an evolutionary theory of adaptive growth. They supposed economic growth as a product of structural change and economic self-transformation based on processes that are closely connected with but not reducible to the growth of knowledge.

The dominant connecting theme is enterprise, the innovative variations it generates and the multiple connections between investment, innovation, demand and structural transformation in the market process. Metcalfe and Foster explored the dependence of macroeconomic productivity growth on the diversity of technical progress functions and income elasticities of demand at the industry level, and the resolution of this diversity into patterns of economic change through market processes. They show how industry growth rates are constrained by higher-order processes of emergence that convert an ensemble of industry growth rates into an aggregate rate of growth. The growth in productivity, output and employment is determined mutually and endogenously, and its value depends on variations in the primary causal influences in the system.

Table 1. Contrast between new growth theories and evolutionary growth

<b>Issues</b>	<b>New Growth Theories</b>	<b>Evolutionary Theories</b>
What is the main level of aggregation?	Aggregate models based on neo-classical micro-foundations (methodological individualism)	Towards a co-evolution between micro-levels and macro-levels of analysis ('non-reductionism')
Representative agent or heterogeneous individuals?	Representative agent and typological thinking	Heterogeneous agents and population thinking
What is the mechanism of creation of innovation?	Learning by doing and searching activity by the R&D sector; radical innovations and General Purpose Technologies	Combination of various forms of learning with radical technological and organizational innovations
What is the dynamics of the growth process? How is history conceived?	History is a uniform-speed transitional dynamics	Towards a combination of gradualist and dynamics: history is a process of qualitative change and transformation
Is the growth process deterministic or unpredictable?	'Weak uncertainty' (computable risk): stochastic but predictable process	'Strong' uncertainty: non-deterministic and unpredictable process
Towards equilibrium or never ending	Towards the steady state	Never ending and ever changing

### 3. 2. Econometric modeling in the evolutionary economic framework

Evolutionary economics in general and evolutionary econometrics in particular are not an arbitrarily choice. It is both relevant and has theoretical foundations. Its relevance is driven by the nature of that which is supposed to be integrated with the previous two papers to form an integrated dissertation. The theoretical basis for such a modeling is drawn from a self-organization approach and analyzed by the logistic diffusion growth model.

Evolutionary economics and the subsequent developments of its estimation techniques have enabled researchers to explore the advantages of evolutionary economics. This methodology is offered to construct an econometric model in the presence of a structural change of an evolutionary type. Evolutionary economics has, in its various approaches, been concerned with economic processes that arise from systems which are subject to on-going structural changes in historical time. Foster and Wild (1999a) identified three characteristics that all evolutionary representations of economic processes seem to share:

- A system that is undergoing a cumulative process of structure building, which results in increasing organization and complexity, cannot easily reverse its structure;
- In the face of this time irreversibility, structure can change in non-linear and discontinuous ways in the face of exogenous shocks, particularly when the relevant evolutionary niche is filled; and
- An evolutionary process of on-going structural change introduces an increasing degree of fundamental uncertainty. Thus, a great deal of structure-building involves the installation of protective repair and maintenance sub-systems.

Based on these arguments, we use a logistic diffusion equation offered by Foster and Wild (1999b) as a theory of historical process. In real terms it is rooted in the Bernoulli Differential Equation of the type shown in the Appendix. The last line in Eq A1 is a Logistic Differential Equation of First Order (LDEFO). Based on Eq A1, Foster and Wild (1999b) have developed an econometric model in the presence of evolutionary change as:

$$(1) \quad \frac{dX}{dt} = b \left( 1 - \frac{X}{K} \right)$$

In Eq 1,  $b$  is the net, that is, it allows for deterioration or deaths, firm entry-exit rate, or diffusion coefficient and  $K$  is the carrying capacity of the environment, for example, total industry or economy's market size, employment or output over which each firm will compete to capture as much of it.  $K$  is a constraint, for example, the total sales of an industry and  $X$  could be a firm's sales so that  $X/K$  is the firm's market share.

Two points must be raised about Eq 1. First  $X/K$  can be understood as any share. If we are to work at the macro-level, we may interpret  $X/K$  as the ratio of GDP to capital stock. This ratio is less than 1 because at any point in time the total national output is some fraction of inputs, the magnitude of the fraction depending on the productivity of the economy.

Eq 1 can be expanded to employ the existing econometric framework for estimation. Foster and Wild (1999b) have acknowledged that the application of the Logistic Diffusion Equation (LDE) of this type has been common in literature on the economics of innovation, following the pioneering work of Griliches (1957). However, economists have tended to view LDE in terms of disequilibrium adjustment from a stable equilibrium state to another in economics of evolutionary growth theory.

As it stands, Eq 1 depicts a smooth process tending towards infinite time. Only in a discrete interval version of the LDE can we generate the kinds of discontinuities that we can see in historical data. However, discrete interval dynamics are not pronounced features of most aggregated economic data. Thus, it is unlikely in most cases that we can generate a discontinuity endogenously.

Now it is convenient for the purposes of an econometric investigation to rearrange Eq 1 in the following way to obtain the Mansfield (1981) variant, employed in many such studies. Dividing both sides of Eq 1 by  $K$  and rearranging, we arrive at:

$$X_t - X_{t-1} = X_{t-1}b \left( 1 - \frac{X_{t-1}}{K} \right) + u_t \text{ approximately,}$$

$$(2) \quad \ln X_t - \ln X_{t-1} = b - bX_{t-1}/K + e_t \text{ where } e_t = u_t/K$$

The transformation into approximation in Eq 2 allows the logistic equation to be estimated linearly and the error term is corrected for bias because of the upward drift of the mean of the X-series.

Eq 2 offers a representation of the endogenous growth of a self-organizing system, subject to time irreversibility and constrained by boundary limits. To come up with the complete econometric model, Foster and Wild qualified their argument in the following ways:

- Regulation in the economic system can restrict economic agents and their organizations to particular market niches. This means, again, that the principle of competitive exclusion is significantly weakened. For example, governments restrict the issue of bank licenses, which preserves a niche which non-bank financial institutions have difficulty entering. Typically, competition in the economic sphere is overlaid by ‘public interest’ regulations that attempt to limit competition;
- Economic sub-systems rely on an interaction with the wider economic system in order to engage in trade. Thus, the  $K$  limit for a particular system will tend to rise continually in line with the general expansion of economic activity; and
- Increasing politicization of an economic system will lead to more predator-prey-type interactions. This will tend to occur in saturation phases of LD growth. Thus, we do not always witness smooth transitions from one LD growth path to another but, instead, Schumpeterian ‘creative destruction’, dominated by conflict and discontinuous dissipation of an accumulated structure (that is, a rapid fall in  $K$ ).

Taking into account these qualifications, the authors arrived at the following LDE which is suitable for application in economics:

$$(3) \quad \ln X_t - \ln X_{t-1} = [b(\cdot)][1 - \left\{ \frac{X_{t-1}}{K(\cdot)} - a(\cdot) \right\}] + e_t$$

Thus,  $b$  and  $K$  are now, themselves, functions of other variables. The function  $b(\cdot)$  allows

for factors that affect the diffusion coefficient, rendering it non-constant over time and  $K(\cdot)$  takes account of factors in the greater system that expand or contract the capacity limit faced by the system in question. The resource competition term,  $a(\cdot)$ , is now a more general functional relationship than the simple mechanism containing, for example, relative prices and existing demand for a particular product, the general economic condition in the environment.

A potential problem with Eq 3 is that, as  $X$  tends to its limit, growth in  $X$  will tend to 0 so that the impact of factors in  $b(\cdot)$  will also tend to 0. This is unlikely to be the case, so it is more appropriate to allow exogenous variables that affect the diffusion rate, to influence the rate of growth of  $X$  with the same strength at all points on the logistic diffusion:

$$(4) \quad \ln X_t - \ln X_{t-1} = [b(\cdot)] \left[ 1 - \left\{ \frac{X_{t-1}}{K(\cdot)} - a(\cdot) \right\} \right] + b(\cdot) + e_t$$

As it stands, Eq 4 could be viewed as a disequilibrium process tending to an equilibrium defined in terms of  $K(\cdot)$  and  $a(\cdot)$ . However, such an equilibrium interpretation differs from that in conventional usage. The non-stationary process modeled by Eq 4 represents neither a mean reversion process in the presence of a deterministic trend, nor a co-integrated association between  $X$  and variables in  $K(\cdot)$  and  $a(\cdot)$ , in the presence of a stochastic trend.

The stationary state to which the logistic trajectory tends is the limit of a cumulative, endogenous process, not a stable equilibrium outcome of an unspecified disequilibrium mechanism following an exogenous shock. The functions  $K(\cdot)$  and  $a(\cdot)$  allow for measurable shocks to the capacity limit and  $b(\cdot)$  encompasses the effect of exogenous shocks which alter the diffusion rate.

One final development is necessary. Although an equilibrium correction mechanism is inappropriate in this type of a model, homeostasis will occur in the short period around what can be viewed as a moving equilibrium.

Eq 4 relates to the momentum of a process and, as such, some path dependence is likely to exist in the sense that the system in question will still have a (decelerating) velocity even if all endogenous and exogenous forces impinging on the system cease to have an effect.

This is likely to be stronger the more non-stationary the variable in question is and the shorter the observation interval. Imposing a simple AR (1) process, we get:

$$(5) \quad \ln X_t - \ln X_{t-1} = [b(\cdot)] \left[ 1 - \left\{ \frac{X_{t-1}}{K(\cdot)} - a(\cdot) \right\} \right] + b(\cdot) + c(\ln X_t - \ln X_{t-1})_{t-1} + e_t$$

In conventional treatments of path dependence in time-series data, constructs such as the ‘partial adjustment hypothesis’, concerning the presumed disequilibrium movements of levels of variables, are used to rationalize the use of lagged dependent variables. Inclusion of a lagged dependent variable requires upward revision of the estimated coefficients on explanatory variables in order to obtain their ‘equilibrium’ values. Here, the interpretation is different, but related. Instead of viewing a lagged dependent variable as evidence of

sluggishness, we view its presence in our growth specification as evidence of momentum in the process (Foster and Wild, 1999b). In Eq 5 we can note that the left hand side is equivalent to the growth rate of series X. In this paper, it could be the growth rate of productivity.

### **3.3 Empirical evidence of evolutionary econometrics**

Empirical literature on evolutionary economics is scarce. However, there are some works which focus on the macro-level, for example, Foster (1992, 1994) and Hodgson (1996).

Foster (1992) looked into a new perspective on the determination of sterling M3 using econometric modeling under the presence of evolutionary change. First he obtained a logistic diffusion model from the first order differential equation. Next he modeled the evolution of M3 in log-linear specification in the form of evolutionary econometrics. He noted Ordinary Least Square (OLS) and Recursive Least Square (RLS) as favored estimation methods in such a condition. He estimated over 1963 to 1988 datasets obtained from the UK monetary authority. He concluded that it was possible to understand the determination of M3 by viewing it as money supply, rather than a money demand, magnitude which is an outcome of a historical process. Such a process has been modeled as institutionally driven and subject to evolutionary change.

In Foster (1994), we can also find an evolutionary macroeconomic approach stressing institutional behavior used for estimating a model for Australian dollar M3. The conclusion is that since Australia and UK have the same cultural and institutional heritage, evolutionary econometrics has captured a similar M3 creation process in both countries implying the appropriateness of an evolutionary approach for studies involving the diffusion process.

The most interesting out of these is Hodgson (1996) as it is the most direct theoretical and empirical research in long-term economic growth. He argues that his work is in part inspired by the work of institutional economics such as Nelson and Winter, Thorstein Veblen (who was the first to suggest the use of economics as an evolutionary analogy taken from biology). His empirical estimation starts by placing major stress on institutional disruptions such as wars or revolutions and on the existence of political institutions such as existence of multi-party systems.

Hodgson used a regression analysis to provide some preliminary empirical validation for his ideas. He admitted that it was not a fully-fledged macroeconomic model, saying that the available data were crude and limited to provide a more ambitious and adequate test. He used real GDP per worker-hour as the index of productivity from Madison's data and summarized his findings as: First, two kinds of disruptions (disruption of extensive foreign occupation of home soil and revolution) seem to be significant in determining and eventually advancing productivity growth. Second, there is evidence that the growth trajectory is determined by the timing of industrialization. Third, a relatively stable international order is found to be significant and positively related to growth.

Another is that of Stockhammer, Onaran and Ederer (2008) who estimated the relationship between functional income distribution and aggregate demand in the Euro area. They modeled aggregate demand as: aggregate demand (AD) is the sum of consumption (C), investment (I), net exports (NX) and government expenditure (G). All variables are in real terms. In their general formulation, consumption, investment and net exports are written as a function of income (Y), the wage share ( $\Omega$ ) and some other control variables (summarized as z). These latter are assumed to be independent of output and distribution. Government expenditures are considered to be a function of output (because of automatic stabilizers) and exogenous variables (such as interest rates). However, as this paper focuses on the private sector, this will play no further role in our analysis. Aggregate demand thus is:

$$AD = C(Y, \Omega) + I(Y, \Omega, z_I) + NX(Y, \Omega, z_{NX}) + G(Y, z_G)$$

Their basic assertion for the inclusion of income distribution into consumption, investment and net export and government expenditure terms is: in the consumption function wage incomes (W) and profit incomes (R) are associated with different propensities to consume. The Kaleckian assumption is that the marginal propensity to save is higher for capital incomes than for wage incomes; consumption is therefore expected to increase when the wage share rises. They argue that Keynesian as well as neo-classical investment functions depend on output (Y) and the long-term real interest rate or some other measure of the cost of capital. The latter is part of  $z_I$ . In addition to output and interest rate, they argue that investment is expected to decrease when the wage share rises because future profits may be expected to fall. Moreover it is often argued that retained earnings are a privileged source of finance and may thus influence investment expenditures.

They claim that first, the policy implication of their findings is that wage moderation in the EU is unlikely to stimulate employment. They suggest that wage moderation leads to a (moderate) contraction in output. Since an expansion in output can be regarded as a necessary (but not sufficient) condition for an expansion in employment, wage moderation (at the EU level) is not an ‘employment-friendly’ wage policy. Their second implication refers to wage coordination; they contend their findings suggest that demand is wage-led in the Euro area. This finding does not extend to individual Euro member states.

This paper takes the advantage of the formalization of evolutionary economics by Foster and Wild (1999) and Foster (1994, 2014).

## 4. Empirical results

### 4.1 The data and variables

The objective of this section is to examine if firms' access to bank loans has any effect on growth through<sup>1</sup> its effects on functional income distribution. The dataset is the medium and large manufacturing industries compiled by the Central Statistical Agency of Ethiopia (CSA). The available panel data covers 1996 to 2009 with 611 and 1,943 firms in 1996 and 2009 respectively.

If access to bank loans first affects functional income distribution and if functional income distribution affects productivity growth that would imply that facilitating access to bank loans might ultimately foster growth of the economy. To achieve this objective, we first explore the real firms over the period on some key variable and econometrically estimate Eq 5 using the Generalized Method of Moments (GMM). Finally alternative policy simulation scenarios are performed to understand the full effect of bank loans, income distribution and productivity growth linkage.

First, from firm-level data, the parameters of interest are computed for each firm for each year. These are:

- Employment share (EMPSHAFIRM): Is supposed to capture if there is an indication of structural change, that is, the movement of labor from less productive to more productive sectors;
- Market share (MKTSHARE): This is the available resource over which firms have to compete. It is through this competition process that decisions to invest on productivity fostering factors are undertaken;
- Output share (OUSHA): Firms can also compete over industry output; and
- Productivity growth (GROWTHPRO): Is the main variable of interest. Its growth rate is understood as the growth of mean characteristics in evolutionary economics. Thus, growth is perceived to mean growth of productivity.

Based on these variables, this paper tries to draw some inferences about the connection between access to bank loans, functional income distribution and productivity growth.

### 4.2. Results from data exploration

The evolution of employment shares, market shares, output shares and growth of productivity are shown in Figures 1-4 respectively. The purpose of these figures is to learn if there is any indication of a structural transformation process within the manufacturing sector. If there is a change in the structure of production in the manufacturing sector, we expect the labor share to be continuously shifting within the industry. The shift should take place from low productivity to high productivity industries. This would mean higher labor productivity and consequently higher labor incomes which will form a positive feedback loop with productivity.

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<sup>1</sup> In evolutionary growth framework, growth is mainly understood as growth of any mean characteristics (in our case productivity growth).

From Figure 1 we observe movements for employment share within the industries only for 11 industries. We identified these industries from the data as:

- Production, processing and preserving of meat, fruit and vegetables
- Manufacture of animal feed
- Manufacture of non-metallic NEC
- Manufacture of basic iron and steel
- Manufacture of other fabricated metal products
- Manufacture of pumps, compressors, valves and taps
- Manufacture of other general purpose machinery
- Manufacture of batteries
- Manufacture of bodies of motor vehicles
- Manufacture of parts and accessories
- Manufacture of furniture

From the firm level dataset, it was possible to learn that most of the firms within these industries had access to bank loans. For example, overall, the 105 firms within the production, processing and preserving of meat, fruit and vegetable industries had access to bank loans. In the manufacture of animal feed industry, out of 98 firms 37 had access to bank loans. Generally, all the indicated firms had access to bank loans during the years of observation. In Figure 1 we can observe that in these industries, there is a significant movement (fluctuation) in employment share. The only exceptions are spinning, tanning and publishing industries in which all firms had access to bank loans. However, any indication of movements in the employment share is not displayed.

One can argue that the employment share must be taking place within the same sector (industries) and not across industries. If the reallocation of labor was taking place across industries, we could have observed variations in the employment share in the rest of the industries, but this is not evidenced.

Whether these industries are high productivity sectors and hence growth and equality promoting is also another area of enquiry. But looking at the face value alone, we may tentatively conclude that in particular those industries related to metallic manufacturing are connected to the government.

Insert Figure 1 about here

Figure 2 displays how market shares in each industry have been evolving. We can observe that market share was almost constant over the observation period. This may tell us of a lack of strong competition among similar firms. The economic reason could be, for example, unsatisfied demand in the goods market.

Insert Figure 2 about here

Referring to Figure 3, firms' shares in total industry output is more pronounced than the market share. This may tell us the underlying market structure which subsequently might have an effect on functional income distribution and productivity growth.

Insert Figure 3 about here

It has been discussed that firms are at the heart of an evolutionary approach to economic growth and growth of productivity at the firm level is a key to economic growth. We can see from Figure 4 that there are fluctuations in the productivity growth rate (from -20 per cent to 10 per cent). We can also note that, for example, the productivity growth for production, processing and preserving of meat, fruits and vegetables remained positive, which might be an indication of the effect of access to bank loans.

Insert Figure 4 about here

### 4.3 Econometric results

This section deals with the econometric estimation of the logistic differential equation in Eq 5. The variables entering the model are of two natured: the evolutionary component and the exogenous component.

We estimated Eq 5 using firm level panel data. To achieve this, the data was transformed (logarithms, growth rates, lags and differences) so that the transformed data was consistent with the evolutionary econometric framework.

The dependent variable is change in the mean characteristics (growth of productivity). The explanatory variables are growth in labor share (GRWTHLSHARE), the complement<sup>2</sup> of the output share (COMPVOUSHA), technically one minus output share to fit the first term in Eq 5, complementary market share (COMPMKTSHARE), again, the same interpretation as before so that it is consistent with Eq 5, lagged change in labor productivity (LAGDELTFP) which represents the last term of Eq 5 and finally, employment share of each firm (EMPSHAFIRM).

For the evolutionary approach, once the logistic differential in Eq 5 is formulated it can be estimated using the standard panel data econometric techniques (random effect, fixed effect or GMM) which do not require separate treatment here. The reported results are with Wald chi-square value of 773.57 with six degree of freedom and probability value of ( $p > \chi^2$ ) of 0.0000 (Table 2).

The estimated result indicates all explanatory variables entered the estimation with statically significant estimates. As expected productivity is positively affected by the growth in labor share. However, the employment share entered with a negative and statistically significant coefficient. We may interpret this as lack of labor movement from low productive to high productive industries.

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<sup>2</sup> Here the complement of variable  $x$  is equal to  $(1 - x)$  (see the first term of the right hand side in Eq 2.5 in Section 4).

Table 2. Estimation Result (GMM): Dependent variable: Growth of productivity.

Variable	Coeff.	Std. Error	z	P>[Z]
GRWTHLSHARE	.00052	0.0001	3.47	0.001
COMPVOUSHA	-5.626	0.409	-13.75	0.000
COMPMKTSHARE	4.251	0.456	9.32	0.000
LAGDELTFP	-0.412	0.0203	-20.20	0.000
EMPSHAFIRM	-4.068	1.556	-2.61	0.009
cons	0.9196	0.421	2.18	0.029

## 5. Summary, conclusions, policy recommendations and future areas of research

The basic research question in this paper was explaining how firm level labor share affects firm and industry level productivity and how it affects aggregate productivity in an economy taking the case of Ethiopia.

The most direct interpretation of the estimated result is that the evolution and change in mean characteristics (change in productivity) are positively affected by the growth of functional income distribution (the growth in labor share: even if the economic sign of the coefficient is of small order), its statistical significance is quite acceptable.

The other variable of interest here is employment share of each firm within an industry, which entered the model with a negative sign but a significant coefficient. In economic terms, the positive and negative coefficients of labor share within a firm and employment share of each firm within the industry tell us very important information about structural change within the manufacturing sector.

If structural change was evident, employment share would have entered with a positive effect. However, it did not do this. Therefore, this does not support the popular view of Structural Bonus Hypothesis which postulates a positive relationship between structural change and economic growth. This hypothesis was based on the assumption that during the process of economic development, economies upgrade from industries with comparatively low to those with a higher value added per labor input. For example, Timmer and Szirmai (2000) have a detailed explanation on this.

Instead, the result is supported by an almost opposite mechanism, where structural change has a negative effect on aggregate growth; this is revealed by Baumol's hypothesis of unbalanced growth. Intrinsic differences between industries in their opportunities to raise labor productivity (for a given level of demand) shift ever larger shares of the labor force away from industries with high productivity growth towards stagnant industries with low productivity growth and accordingly higher labor requirements. In the long-run, the structural burden of increasing labor shares getting employed in the stagnant industries tends to diminish the prospects for aggregate growth of per capita income. Baumol (1967) is key literature on this.

when the complement of firms' market share enters the regression result with a positive sign, the actual market share would have entered with a negative sign which has a direct and clear economic meaning, that is, since firms may try to capture the market through nominal ways (for example, price competition, or advertising, or any other institutional arrangements) this will harm productivity. Our major conclusion is lack of strong evidence for intra-industry selection.

The policy lesson that we learn is that access to bank loans is of great the importance to firms. Particularly in those industries (spinning, tanning and publishing industries) in which all firms had access to bank loans have revealed movements in the employment share, which is evidence of structural transformation.

There are reasons why it is important to introduce appropriate public loan policy, i.e., ensuring lending channel of monetary policy to work without breaks. First, a credit aggregate can be a better indicator of monetary policy than an interest rate or a monetary aggregate in Ethiopia. Second, a monetary tightening that reduces loans to firms can have negative distributional consequences. Particularly for those firms for whom bank loans are a primary source of finance, ease of access to bank loans can have economy wide distributional consequences. More specifically, the credit policy should be such that manufacturing firms get better access to bank.

It is desired that the future research direction includes economy wide modeling, estimation and more formalization of evolutionary economic models to study the link between accesses to bank loans and its effects on income distribution and inclusive economic growth.

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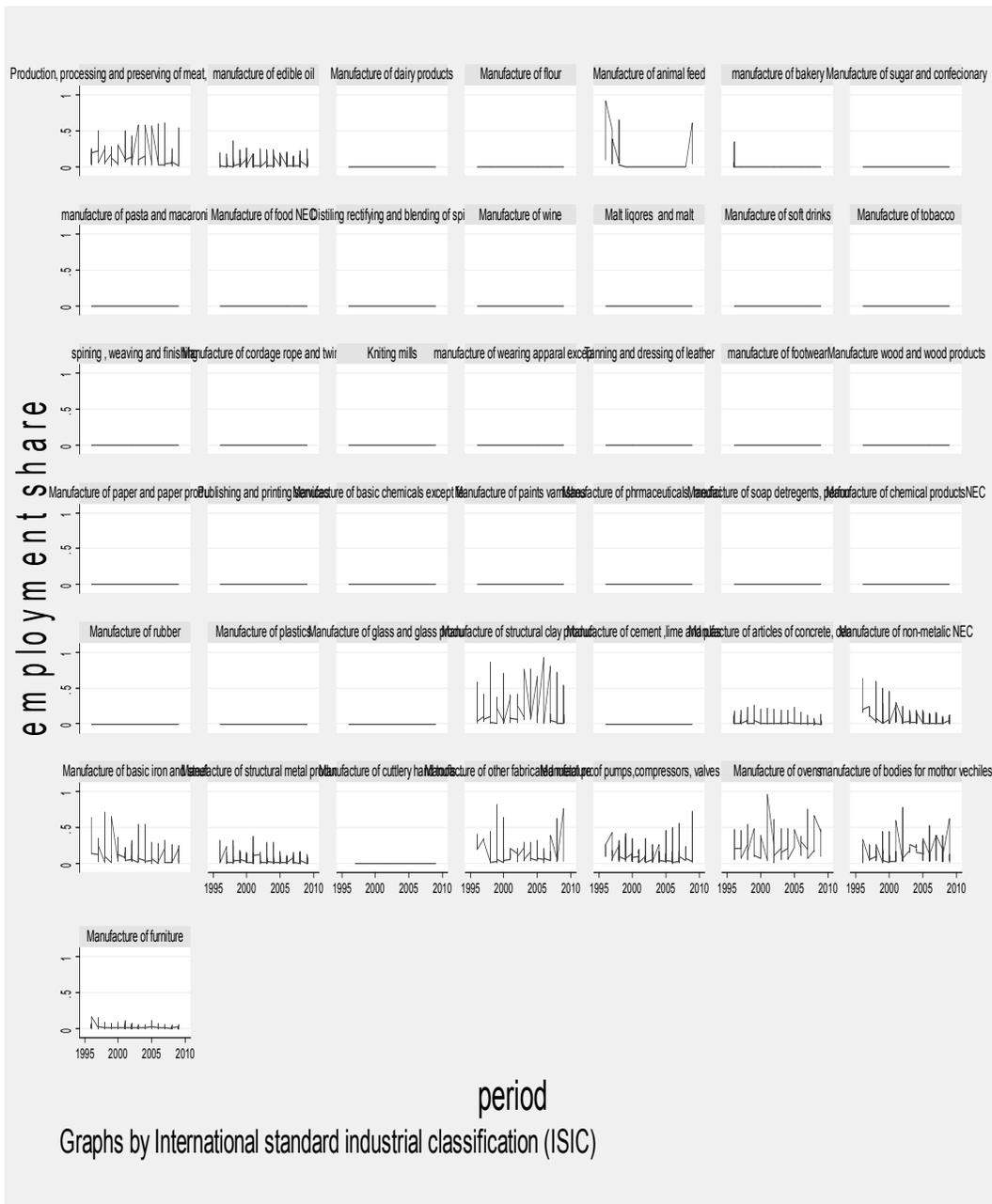


Figure 1. Evolution of employment share

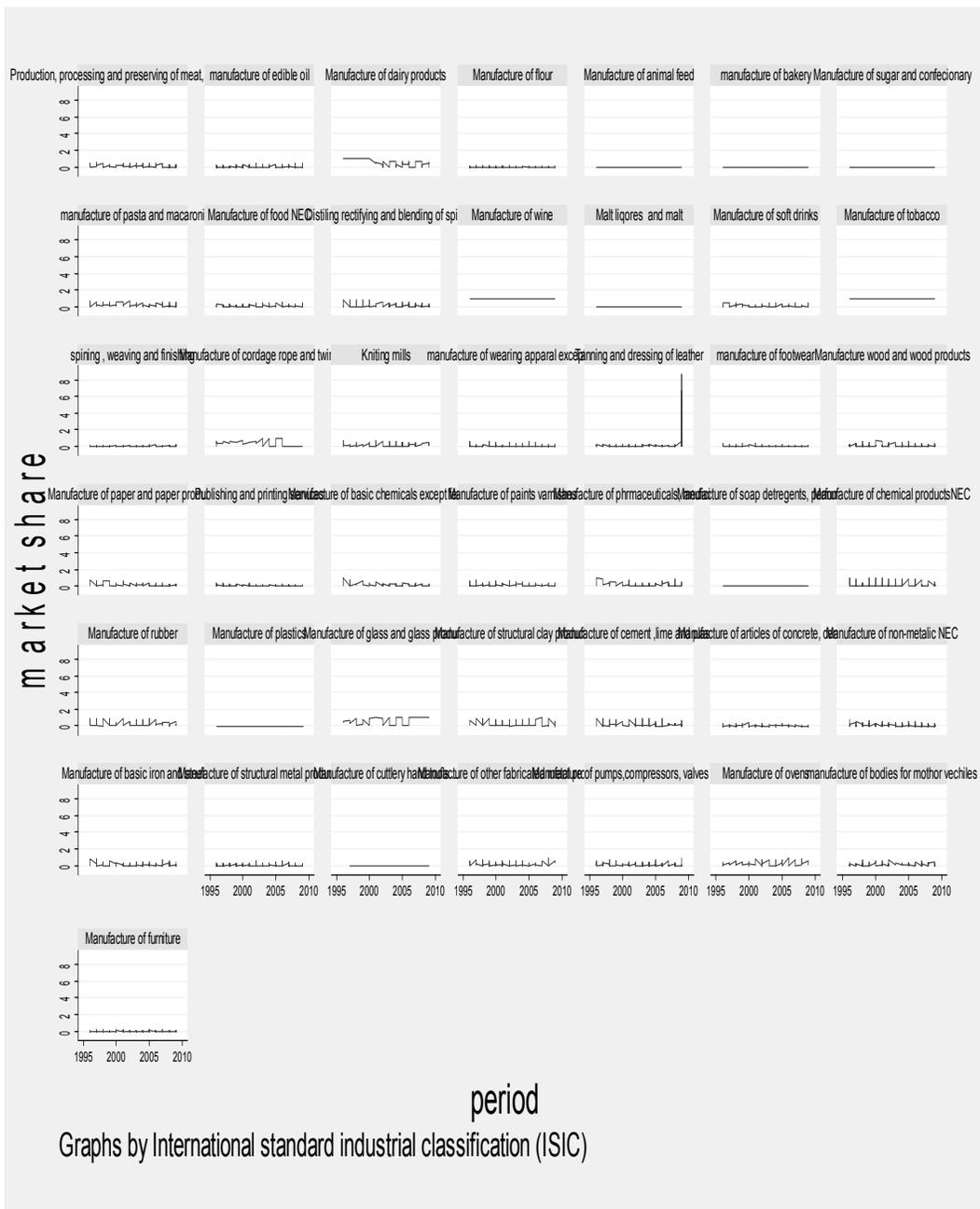


Figure 2. Evolution of market share

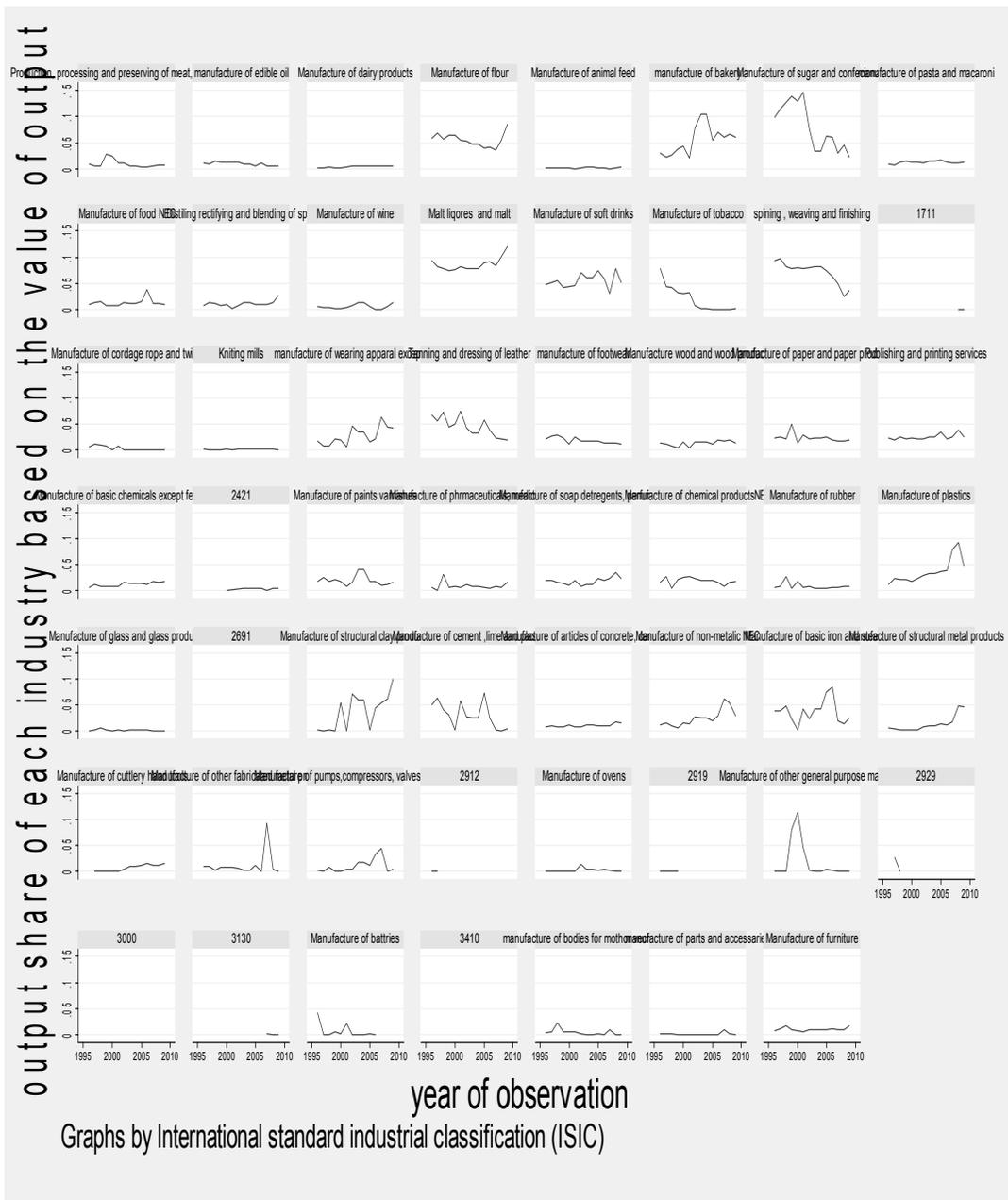


Figure 3. Evolution of output share at the industry level

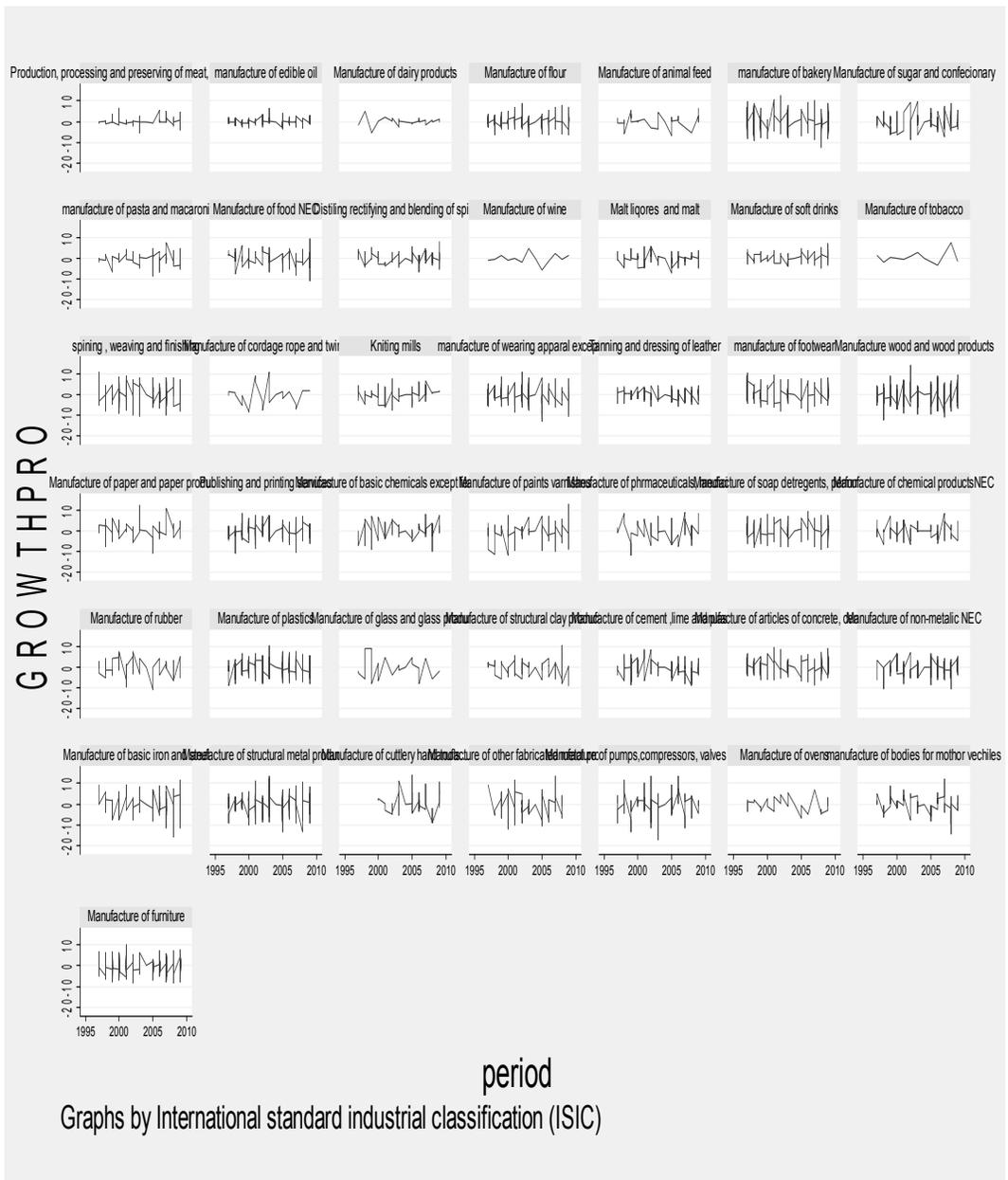


Figure 4. Evolution of productivity growth

**Appendix A:**

$\dot{X} + a(t)X = b(t)X^r$ , if  $r = 1$ , it is easily separable and becomes

$\dot{X} + a(t)X = b(t)X^r$  and introducing  $Z = X^{1-r}$

$$\dot{Z} = (1-r)X^{-r} \dot{X}$$

$$\text{But } \frac{\dot{X}}{X} + a(t) = b(t)X^{r-1} \Rightarrow \dot{X} = (b(t)X^{r-1} - a(t))X$$

Therefore,

$$\dot{Z} = (1-r)X^{-r} \dot{X} = (1-r)X^{-r} (b(t)X^{r-1} - a(t))X$$

$$(A1) \quad \dot{Z} + (1-r)a(t)Z = (1-r)b(t)$$